Errata Corrige for "Fundamentals of Applied Functional Analysis"

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The *Errata Corrige* addresses to all readers of the monograph by D. Mitrović and D. Žubrinić: *Fundamentals of Applied Functional Analysis*, Pitman Monographs and Surveys, Addison Wesley Longman Limited, 1998. It is now distributed by the CRC Press. The list of corrections has been prepared, contrary to our hopes, before issuing the second enlarged and corrected edition of our book, which does not seem to be very near.

Authors, Zagreb, 2005.

Notation: by 3+13 we mean line 13 on p. 3, and by 3-5 we mean line 5 from below on p. 3.

Chapters 1, 2, 3

3+13 change "the so-called G_δ-set" to "a so-called G_δ-set"

3+14 change "the so called F_δ-set" to "a so-called F_δ-set"

4+14 change "a nonempty set X" to "of a nonempty set X and $\overline{\mathbf{R}} = \mathbf{R} \cup \{\infty, \infty\}$"

8+1 change "$\mu^*(A)$" to "$\mu^*(E)$"

8 in Theorem 1 delete "complete"

9+12 after "a negligible subset is measurable" to add "The restricition μ in Theorem 1 is a complete measure."

9+15+16 in Proposition 4 change "the algebra" to "the σ-algebra"; change "by an outer measure" to "by the outer measure";

9. Example 2+6 change "Let λ be an extended" to "Let λ be the extended"

10-4 change "$\mathcal{S}$ be an algebra" to "$\mathcal{S}$ be the algebra"

11+2 change "mutually disjoint" to "mutually non-overlapping"

11+17 change "We arrive to the notion" to "We arrive again to the notion"

12. Example 1+3 change $u = 1$ to $u = \frac{1}{3}$

13-5 change "the complete" to "a complete"

15 Theorem 2+2 change Prove to Then

16+5 change "the space of Borel measure on X" to "a space of Borel measure space on X"

16+10 change "measurble" to "measurable"

16+11+12+13 change "if and only if the set E is measurable" to "if and only if $\mu(E) < \infty$"

16+11+12+13 change "Any constant function defined on X is simple" to "Any constant function defined on X with $\mu(X) < \infty$ is simple"

21-11-12-13-14-15 change "from E into $\mathbf{R}$. For every $\varepsilon > 0$ and $n \in \mathbf{N}$ we consider" to "from E into $\mathbf{R}$. For the completeness of the Section, for every $\varepsilon > 0$ and $n \in \mathbf{N}$ we consider again"

28 Theorem 3 change "integrable functions" to "integrable functions from X into $\mathbf{R}$"

31 Exercise 1, c) In Solution on p. 361 change $\int_0^1 u_n(x)\,dx = \frac{n}{(n+1)^2}$ to $\int_0^1 u_n(x)\,dx = \frac{n}{(n+1)(n+2)}$

34+1+2 Theorem 1 change "with a σ-finite measure" to "with σ-finite measures"

35-2 change $\frac{xy}{x^2+y^2}$ to $\frac{xy}{(x^2+y^2)^2}$

37 Theorem 1+2 change "and both integrals" to "the two integrals"

39+7 change "Both integrals" to "The two integrals"

40-11 change "a vector space" to "the vector space"

43+11 change "the volume" to "the volume measure"

44+4+7+8 change f, g to u, v four times

44+8 change "both converge or both diverge" to "both are finite or both are infinite"

48+18 change "We shall see in Section 4 that $C_0^\infty(\Omega)$ is dense in $L^p(\Omega)$" to "We shall see in Section 4 that $C_0^\infty(\Omega)$, defined in Section 3, is dense in $L^p(\Omega)$"

48 Example 8 The function χ_K is defined on page 14 and redefined on page 49

49 Theorem 1 change (Riesz) to (F. Riesz), and change $\|\varphi\|$ to $\|F\|$

50+1 change "with compact support" to "With compact support (defined in Section 3)"

55-11 change "defined" to "considered"

51-12 change "k times differentiable" to "k times continously differentiable"

52+6 change "length of multiindex" to "length of the multiindex"

52-5-4 delete "provided $v \in C_0^k(\Omega)$"

Chapter 4

62+2 change "For example" to "For instance"

62+4+5+6 change "Thus the equation $u' = \frac{1}{x}$, that is $xu' = 1$ has no solution on $\mathbf{R}$, but only the distribution solution $u(x) = \log|x| + C_1 H(x) + C_2$, where H is the Heaviside function and C_1, C_2 arbitrary real constants" to "Thus the classical solution of the equivalent equation $(*)$ $u' = \frac{1}{x}$ and $(**)$ $xu' = 1$ on $\mathbf{R} \setminus \{0\}$ is $u(x) = \log|x| + C$, where C is an arbitrary constant. This function is the distributional solution on $\mathbf{R}$ of the equation $u' = \mathrm{VP}\,\frac{1}{x}$ since $\log|x|$ is a distribution on $\mathbf{R}$ (Note that the distribution $\mathrm{VP}\,\frac{1}{x}$ coincides with the distribution $\frac{1}{x}$ on $\mathbf{R} \setminus \{0\}$). The distributional solution of $(**)$ on $\mathbf{R}$ is $u(x) = \log|x| + C_1 H(x) + C_2$, where C_1, C_2 are arbitrary constants (p. 69, Example 10, p. 92)."

62-9 change "limit (accumulation) points" to "boundary points"

65-18 change "there exists a compact set" to "there exists a fixed compact set"

66-5 change $\psi(x) = \frac{1}{m}(x)$ to $\psi(x) = \frac{\varphi(x)}{m}$

67-4 After $u(x) \equiv 0$. write (The zero distribution on $\Omega \subseteq \mathbf{R}^N$ is defined in Section 4.4.)

72-13 change "as an ordinary function" to "as a point function"

72-6-5 change "its closure $\overline{K}$ lies in Ω" to "its closure $\overline{K}$ is compact in Ω"

72-6 change "a subspace of $\mathcal{D}'(\Omega)$, $\Omega \subseteq \mathbf{R}^n$" to "a subspace of $\mathcal{D}(\Omega)$, $\Omega \subseteq \mathbf{R}^n$

73 Theorem 6, 8 change "the norm" to "a norm"

81 Proposition 1 change $D^\alpha \mathbf{R}$ to $D^\alpha T$

83+6 change $(\langle T, \varphi \rangle)$ to $|\langle T_k, \varphi \rangle|$

84-1, Exercise 3.(4) at the end to add: for the definition of convergence see Section 11, p. 154.

87+11+12 change "a_α" to "the a_α" and "the differential" to "a differential"

87+13 change "that L is the differential" to "that L is a differential"

89+1 change $\Delta'(\mathbf{R}^n)$ to $\Delta'(\mathbf{R})$

89+11+12 change $-\langle T, \chi \rangle = \langle T, \chi' \rangle = \langle T, \psi \rangle$ to $-\langle T, \chi \rangle = -\langle T, \psi' \rangle = \langle T', \psi \rangle$

89-15 change "any function φ_0" to "choose a fixed function φ_0"

89-6 change "Using (5) we can define" to "Now we extend"

89-5 change "$\langle T, \varphi \rangle = \lambda \langle T, \varphi_0 \rangle + \langle T, \chi \rangle$" to "$\langle T, \varphi \rangle = \lambda C + \langle T, \chi \rangle$ where C is an arbitrary complex constant and $\chi = \psi'$"

89-3 change "$\langle T, \varphi \rangle = \lambda \langle T, \varphi_0 \rangle - \langle S, \psi \rangle$" to "$\langle T, \varphi \rangle = \lambda C - \langle S, \psi \rangle$"

90+1 delete "where $c = \langle T, \varphi_0 \rangle$"

90+11 change "$\lambda\langle T_1, \varphi_0\rangle$" to "$\lambda C_1$" and "$\lambda\langle T_2, \varphi_0\rangle$" to "$\lambda C_2$"

90+13 change $\lambda\langle T_1 - T_2, \varphi_0\rangle$ to $\lambda(C_1 - C_2)$

90+14 change $C = \langle T_1 - T_2, \varphi_0\rangle$ to $C = C_1 - C_2$

92-4 change "arbitrary real constants" to "arbitrary constants"

93+12 change "Schwartz [1], pp 23, 58" to "in Schwartz [1] pp 123, 282"

94 delete the lines 11 and 12 (concerning the compactness of K)

94+6 change $\operatorname{supp}(T) \subset\subset \mathbf{R}^n$ to $\operatorname{supp}(T) \subset \mathbf{R}^n$

97-6 change $T_k \in \mathcal{E}(\mathbf{R}^N)$ to $T_k \in \mathcal{E}'(\mathbf{R}^n)$

98-14 change $L \in \mathcal{E}(\mathbf{R}^N)$ to $L \in \mathcal{E}'(\mathbf{R}^N)$

98-8 change "$\mathcal{E}(\mathbf{R}^N)$ is L" to "$\mathcal{E}'(\mathbf{R}^N)$ is L"

98 (i) replace the lines from 98-7(i) to 99+2 by "We argue by contradiction. Assume that K is not bounded so that T does not equal to zero on the set $\{x \in \mathbf{R}^2 : |x| > m\}$ for every $m \in \mathbf{N}$. In other words, there is a sequence of functions $\varphi_m \in \mathcal{D}(\mathbf{R}^N)$ with support in $|x| > m$ such that $\langle T, \varphi_m\rangle \neq 0$ for every m. From this fact one can derive $\langle T, \varphi_m\rangle = 1$. But then $\varphi_m \to 0$ in $\mathcal{E}(\mathbf{R}^n)$ as $m \to \infty$ since on every given compact set we have $\varphi_m = 0$ provided m is sufficiently large. Thus T being continuous, we obtain that $\langle T, \varphi_m\rangle \to 0$, which contradicts $\langle T, \varphi_m\rangle = 1$. This shows that K is bounded.

99-6 change "the term" to "the number (or integer)"

100-11 change "in technical sciences" to "in technical sciences (see L. Hörmander [1])"

102+17 change $L^1(\Omega)$ to $L^1(\mathbf{R})$

102-7 change "the limit" to "a limit"

105-8 change $e^{-2\pi t^2}$ to $e^{-\pi t^2}$

110-11-12 change dy to dx in (2) and change $\langle V_y, \varphi(x, y)\rangle$ to $\langle V_y, \varphi(x, y)\rangle\, dx$

113-12 change $u(x)\langle T_y, \varphi_2(y)\rangle$ to $\varphi_1(x)\langle T_y, \varphi_2(y)\rangle$

113-15-14 change "it is necessary to verify its commutativity: $S \otimes T = T \otimes S$, which is included in the definition" to "it is necessary to verify that the tensor product is commutative: $S \otimes T = T \otimes S$, that is, $\langle S_x, \langle T_y, \varphi(x, y)\rangle\rangle = \langle S_y, \langle T_x, \varphi(x, y)\rangle\rangle$

113-2 change $\mathcal{D}(\mathbf{R}^m)$ to $\mathcal{D}(\mathbf{R}^n)$

114-18 change "in the "cuboid" Ω" to "in the "cuboid" "

117-20 change $\langle S * T, \varphi(x, y)\rangle = \langle S * T, \alpha(x, y)\varphi(x + y)\rangle$ to $\langle S \otimes T, \varphi(x, y)\rangle = \langle S \otimes T, \alpha(x, y)\varphi(x, y)\rangle$

117-8 change $\varphi(x + y)$ to $\varphi_k(x + y)$

121+4 change “tensor product” to “convolution”

121-2-3 change $\delta f(0)$ to $\delta'(0)$ and δ^{m-2} to $\delta^{(m-2)}$

123-2-1 change $\delta f(0)$ to $\delta'(0)$ and $\delta^{m-2)}$ to $\delta^{(m-2)}$

126+5 change $(\int_\varepsilon^a r\log r\,dr)$ to $(\int_\varepsilon^a r|\log r|\,dr)d\theta$ and change $\sqrt{\varepsilon}$ to $\sqrt{e}$

131-15 change tempred to tempered

132-16, Example 2 change $|f(x)|$ to $|\varphi(x)|$ in the integral over $\mathbf{R}^n$

133-2 change complexs to complex

133 replace the lines from 133+9 to 133+13 by “Note that $\mathcal{S}(\mathbf{R}^n)$ is a complete space in which all Cauchy sequences converge

137+5 change “In the above proof” to “At the top of previous page.”

137+11 change Theorem 1 to Theorem 4.

137+11, (ii) change “a linear bounded operator” to “an isometric linear operator”

139+2, Theorem 6 change “a unitary operator” to “an isometric operator”

142-10-11 change “The weak and strong topologies on $\mathcal{S}'(\mathbf{R}^n)$ are not equivalent” to “The weak and strong convergence of sequences in $\mathcal{S}'(\mathbf{R}^n)$ are equivalent”

144-3 change $f\varphi$ to fT

147+10 change $\langle T_u, \varphi\rangle$ to $\langle T_u, \hat{\varphi}\rangle$

149+5 change $\mathcal{F}(T^\alpha$ to $\mathcal{F}(T^\alpha)$

150-14 change “As we know (see Example 5)” to “In other words”

151+2 change identical to identity

152+3 change $\check{\varphi}(t) := \mathcal{F}(u)$ to $\hat{\varphi}(t) = \overline{\mathcal{F}}(u)$

155-3 delete Exercise 1

155+7 change $\log(R + i\pi)$ to $(\log R + i\pi)$

157+2+3 change “the contour” to the contour L”

158-10 change “proved that” to “proved the following”

158-6 change “Namely” to “Then”

159-8 change (1915–) to (1915–2002)

Chapter 5

160-1 the equality of integrals over Ω is denoted by (2)

161-16 delete (2)

163+3 change "any point" to "every point"

165+7, Theorem 2 change derivative to derivatives

167+4+5 change $L^1_{loc}(O)$ to $L^1_{loc}(\Omega)$ and $L^p(\mathbf{R}^n)$ to $L^p(\Omega)$

169-7 change "if and only if u is Lebesgue measurable" to "if and only if u is in $\mathcal{S}'(\mathbf{R}^n)$"

171+6 change $L^2(\Omega)$ to $L^2(\mathbf{R}^n)$

171+9 change "Much more" to "A much more"

176+7 change "a domain" to "the domain"

183, Theorem 3 change

$$\frac{\partial u^-}{\partial x_i} = \begin{cases} 0 & \text{if } u(x) < 0 \\ -\frac{\partial u}{\partial x_i} & \text{if } u(x) \geq 0 \end{cases}$$

to

$$\frac{\partial u^-}{\partial x_i} = \begin{cases} -\frac{\partial u}{\partial x_i} & \text{if } u(x) < 0 \\ 0 & \text{if } u(x) \geq 0 \end{cases}$$

185+6 change "x_n-axes" to "x_n-axis"

187-6 change "Kesavan [1]" to "Kesavan [1] (pp. 72–80)"

190+12 change "The above result" to "Theorem 6"

191-14-13 In the inequality change $\geq$ to $\leq$ and minimizing to maximizing

191-7 change $=$ to $\geq$ in the inequality

194-14 change "of case c)" to "of case a)"

196+13+14 change L^p to $L^{p'}$

199+18+22+24 change "well ordered" to "totally ordered" and "well ordering" to "totally ordering"

201, Remark 2 change $s = 1$, $t = 2$ to $r = 1$, $s = 2$

200+16 change sufficiantly to sufficiently

200+13 change $V|_{\mathbf{R}^n\setminus\Omega} = 0$ to $V|_{\mathbf{R}^n\setminus\overline{\Omega}} = 0$

210-13 change coercive to coercive defined on p. 214 (also see pp. 275, 281, 289)

211+3 change $W^{\theta,p}$ to $\widetilde{W}^{\theta,p}$

212-10 change A to C and B to D

Chapter 6

210+16 the notion of "coercive" is defined in 214+9 (see pp. 210, 275, 281, 289)

216-6 change "a given function" to "a continuous function"

229+13+14 change "in the interior" to "in the inner part"

231-6-5 delete "We shall use consecutively Vainberg's theorem, the imbedding result of of Sobole vspaces, and the regularity of solutions of linear elliptic equations."

231-1 a, b positive real numbers

235-12 Example 2 change it to the following: "Let $u : \mathbf{R}^2 \setminus \{0\}$ be defined by $u(x,y) = \mathrm{Re}\,(e^{1/z^4})$ for $z = x + iy \neq 0$. Since the involved complex-valued function is analytic in the punctured plane $\mathbf{C} \setminus \{0\}$, the function u is harmonic in $\mathbf{R}^2 \setminus \{0\}$, that is, $\Delta u = 0$ identically here (Note that a real-valued function in $\Omega \subset \mathbf{R}^2$ is harmonic if and only if it is the real part of an analytic function in Ω. Such a result does not exist in $\mathbf{R}^n$, $n > 2$).

236-4 Theorem 2 change $\inf_\Omega u$ to $\inf_{\partial\Omega} u$

242 Remark 5 change Rayligh to Rayleigh

243-8 change "with the zero" to "with zero"

243-2 change $\xi^2 \|\varphi_i\|^2$ to $\xi_i^2 \|\varphi_i\|^2$

243+4 change Rabiar to Rabier

250+1 change read to pronounced

250+6 change dense to total

251 Theorem 2 where λ_1 is the first eigenvalue of $-\Delta$

253+2 change $=$ to $\leq$

Chapter 7

260, Lemma 1(i) change $\frac{1}{\lambda - \lambda_{k-1}}$ to $\frac{1}{\lambda - \lambda_k}$

261, Remark 2 change "Example 6.1" to "Example 7.1"

271-15 change $\Phi(u_k) \to \Phi(u)$ to $\Psi(u_k) \to \Psi(u)$

272-5 change $\delta(h)$ to $\delta(\varphi)$

274+6 change $\langle u - T_u, h\rangle$ to $\langle u - T_u, \varphi\rangle$

277+6 change $W \ni v$ to $V \ni v$

278, Exercise 3 change "An operator" to "The operator"

278, Exercise 4 change "an operator" to "the operator"

281+4, Theorem 3 change “onto” to “surjective”

282, Theorem 5 change “onto” to “surjective”

282-4 change “countable basis” to “countable total set”

285-17 change “Theorem 1.3” to “Theorem 2.3”

285, Exercise 11 change onto to surjective

287+4 change $\lambda_k \int_\Omega uv\,dx$ to $\lambda_k \int_\Omega u\varphi\,dx$

287-11 change (Theorem 1.4) to (Theorem 2.4)

288-3 change “the fixed point” to “a fixed point”

289-13 change $u = u + w$ to $u = v + w$

290-3 change $V_\varepsilon(\mathcal{K})$ to $T_\varepsilon(\mathcal{K})$

292, Lemma 1, line 5 for $w_1, w_2 \in W$

293+1 change Theorem 2.5 to Theorem 3.5

293+15 change δ^α to $\delta^{\alpha-1}$

296+13 change “Proposition 1.2” to “Proposition 2.2”

300-4-5 change inequalities on the right-hand sides to

$$\geq \mu_- \int_\Omega u\varphi_1\,dex - c\int_\Omega \varphi_1\,dx + t$$

$$\geq \mu_+ \int_\Omega u\varphi_1\,dex - c\int_\Omega \varphi_1\,dx + t$$

300-1-2 change $\int_\Omega u\varphi\,dx$ to $\int_\Omega u\varphi_1\,dx$ twice

300+10 change $\Phi'(u_k) \to 0$ to $\Phi(u_k)v \to 0$ for all $v \in X$

304-12 change Clarke to Clark

304, Theorem 1 change “for every $\overline{\varepsilon}$ there exists $\varepsilon \in (0, \overline{\varepsilon})$ and a deformation” to “there exists $\overline{\varepsilon} > 0$ such that for every $\varepsilon \in (0, \overline{\varepsilon})$ there exists a deformation”

307, Theorem 1 change “there exists $e \in X$, $0 < r < \|\varphi\|$ such that” to “there exists $e \in X$ and an r such that $0 < r < \|\varphi\|$ for which”

308+4 change Thererfore to Therefore

311, Exercise 5, line 2 change “as well” to “either”

Chapter 8

312-1-2 change "mean value theorem" to "intermediate value theorem"

315+10 change "cube U" to "cube in U"

320+1 change Rieman to Riemann

321-8 change brushed to combed

323+18 change "around y_0" to "around v_0"

325+6 change "an infinite" to "the infinite"

325-10 change "was introduced by Schauder" to "were first introduced by D. Hilbert and F. Riesz"

328, Exercise 6, line 2 change "onto" to "surjection"

329+15 change $-\frac{c}{\lambda_1^{1/2}}\|u\|_{H_0^1}$ to $-\frac{c}{\lambda_1^{1/2}}\|u\|_{H_0^1}^2$

331+5 change $(I-L, B_r(u_0), 0)$ to $(I-L, B_r(0), 0)$

332+3+4+9 change u_0 to 0 throughout

334-1 change (μ_0, u_0) to (μ_0, u)

335+7 change $\frac{\partial F}{\partial u}$ to $\frac{\partial F}{\partial u}(\mu, 0)$

336-13 change $(H_1, I_\pm \times B_r(0), (0,0))$ to $(H_1, (I_\pm - \mu_\pm) \times B_r(0), (0,0))$

338, Example 4, line 6 change (x^2+y^2) to $(x^2+y^2)^2$

Appendix

339+11 change "to its disjoint" to "into disjoint"

339-7 change "projection" to "surjection"

340+1 change A to its italics

340+22 change "a partitive set" to "the partitive set"

341+12 change "i (c)" to "and (c)"

341+18 delete open

341-1-2 change "Let $A \subseteq X$. The set is dense in the space (X, τ) if for every $u \in X$ there exists a sequence (u_n) in X such that $u_n \to u$." to "Let X satisfy the first axiom of countability (that is, any point in X possesses a countable base of neighbourhoods and $A \subseteq X$). Then the set A is dense in the space (X, τ) if for every $u \in X$ there exists a sequence (u_n) in A such that $u_n \to u$ in τ."

342+5 change "or a subspace topology" to "or the topology induced by τ in A"

342+16 change “the inverse of every” to “the inverse image of every”

343+7 chage Tietz to Tietze

345-12 change “a complete metric space in X” to “in a complete metric space X”

345-2 change “the convergence” to “A convergence”

347+7 change linear to additive

347+23 change “of linear” to “of compact linear”

348+19 change “its bidual” to “their bidual”

348-10 change “a normed space” to “the normed space” and “separablle” to “separable”

348-11 after “(Riesz’s theorem)” to add “Here we allow Borel measure to change sign, i.e. $\mu(A) \in \mathbf{R}$.”

349-11 change “its dense subspace” to “a dense subspace”

350+14+15 delete lines 14 and 15

351+2 change A_a to A

353, Hilbert space, line 3 change “both variables (obtained by fixing one of the variables)” to “each variable (by fixing the other one)”

354+1 change “an orthogonal sequence of unit vectors” to “a sequence of pairwise orthogonal unit vectors”

355, Theorems about integrals, line 2 change “this book” to “this book.”

Solutions and hints

360; 2.4.1, 361+7 change $n/(n+1)^2$ to $n/(n+1)(n+2)$

365; 4.3.13 change $\delta^{(m)}$ to $\delta^{(m-2)}$

370; 5.8.9 change $f \in H^{-s}(\Omega)$ to $f \in H^{s}(\Omega)$

375; 7.3.12 delete “(we believe that $X = \ell_2$)”

381; 8.2.13 change Tietz to Tietze

382; 8.4.2 change Shauder to Schauder